

# VALIDATION OF GOCE-BASED GRAVITATIONAL GRADIENTS GRIDS BY SPECTRAL COMBINATION METHOD

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## Content:

- Motivation,
- Spectral combination method,
- Numerical experiment,
- Results,
- Conclusion,



# Motivation:

- grids of gravitational gradients produced by three groups  
(see, ESA-funded GOCE+ GeoExplore project, GOCE+ GeoExplore II project or Space-wise GOCE products),
- a method for recovering gravitational field quantities on the Earth surface,
- validation of GOCE-based gravitational grids.

## ESA GOCE Virtual Archive

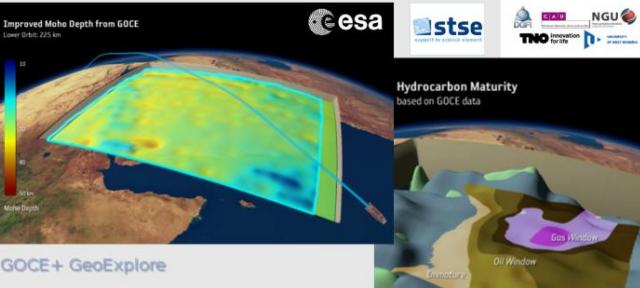
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### GOCE+ Gravity Gradients and GRIDS

Please visit the [GOCE+ GeoExplore data webpage](#) on GOCE Earthnet portal for documentation and latest news.

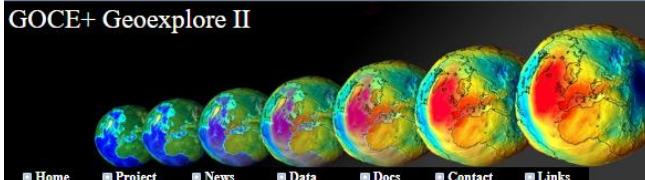


### GRIDS

Full Gravity Gradient grids, at 225 and 255 km.

GGG_225_0003	Computed from GOCE/GRACE gradients lower orbit phase February 2010 - October 2013
GGG_255_0003	Computed from GOCE/GRACE gradients lower orbit phase February 2010 - October 2013
TGG_225	Computed from GOCE/GRACE gradients lower orbit phase August 2012 - October 2013 with topographic correction
TGG_255	Computed from GOCE/GRACE gradients lower orbit phase August 2012 - October 2013 with topographic correction

### GOCE+ Geoexplore II



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### Welcome to our project pages

These pages are web pages for ESA supported project Towards a better understanding of the Earth's interior and geophysical exploration research "GOCE+ Geoexplore II"

### Consortium

The project will be performed by a consortium led by University of West Bohemia (Czech Republic). The consortium consists of six institutes from six European countries (all ESA member states):

UWB - University of West Bohemia, Department of Mathematics, Czech Republic  
AAS - Austrian Academy of Sciences, Space Research Institute, Austria AUT - Aristotle University of Thessaloniki, Department of Geodesy and Surveying, Greece DIAS - Dublin Institute for Advanced Studies, Geophysics Section, Ireland GIS - University of Stuttgart, Institute of Geodesy, Germany TUD - Technical University Delft, Astrodynamics and Space Missions, Netherlands.



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### GOCE Gravity Field Models and Grids

#### GOCE Gravity solution GRIDS

##### Gridded Gravity gradients and anomalies at ground level

[GO\\_CONS\\_GRC\\_SPW\\_2\\_20091101T000000\\_20111231T235959\\_0001.TGZ](#)  
[GO\\_CONS\\_GRC\\_SPW\\_2\\_20091101T055147\\_20120731T222822\\_0001.TGZ](#)  
[GO\\_CONS\\_GRC\\_SPW\\_2\\_20091101T055226\\_20131020T033415\\_0002.TGZ](#)

Show Model characteristics

##### Gridded Gravity gradients and anomalies at satellite height

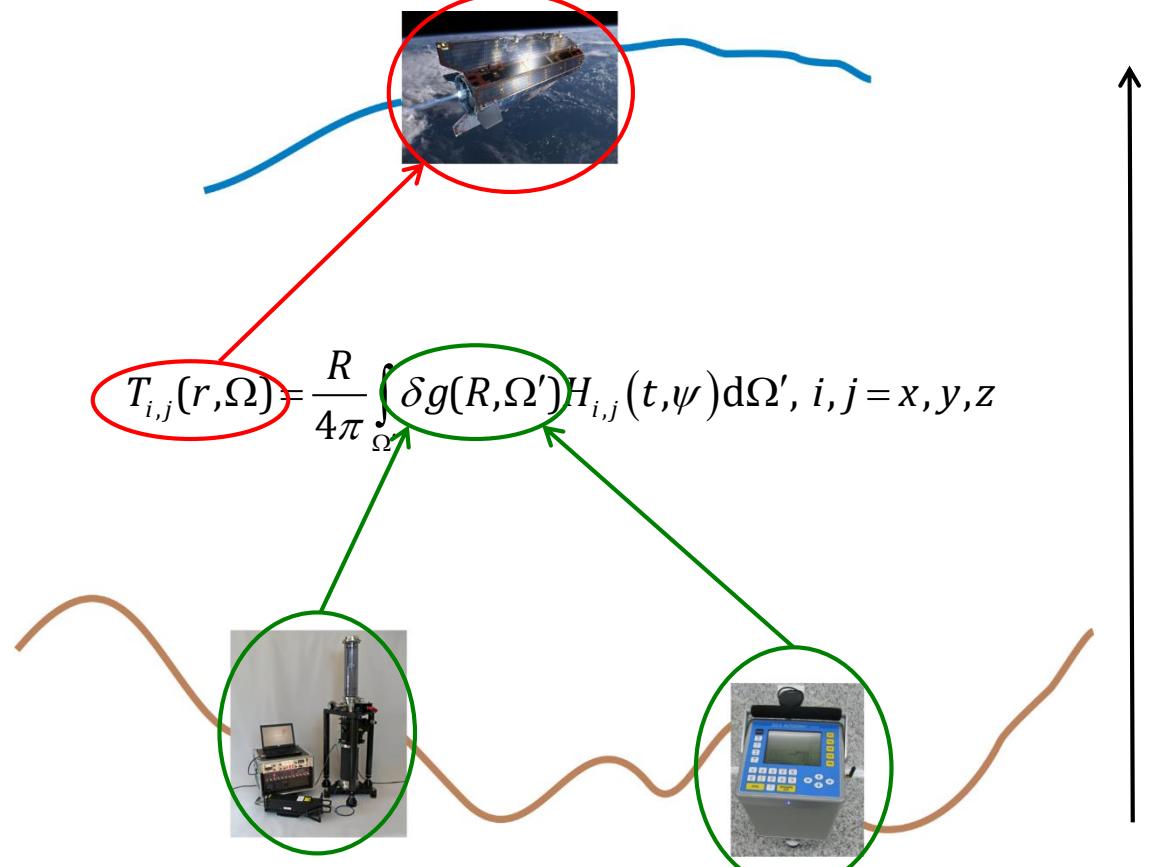
[GO\\_CONS\\_GRD\\_SPW\\_2\\_20091101T055147\\_20100630T180254\\_0001.TGZ](#)  
[GO\\_CONS\\_GRD\\_SPW\\_2\\_20091101T055147\\_20120731T222822\\_0001.TGZ](#)  
[GO\\_CONS\\_GRD\\_SPW\\_2\\_20091101T055226\\_20131020T033415\\_0002.TGZ](#)

Show Model characteristics

# Motivation: Classical approach to validation

Upward continuation:

$$dT_{i,j} = T_{i,j}^{GOCE}(r, \Omega) - T_{i,j}(r, \Omega)$$

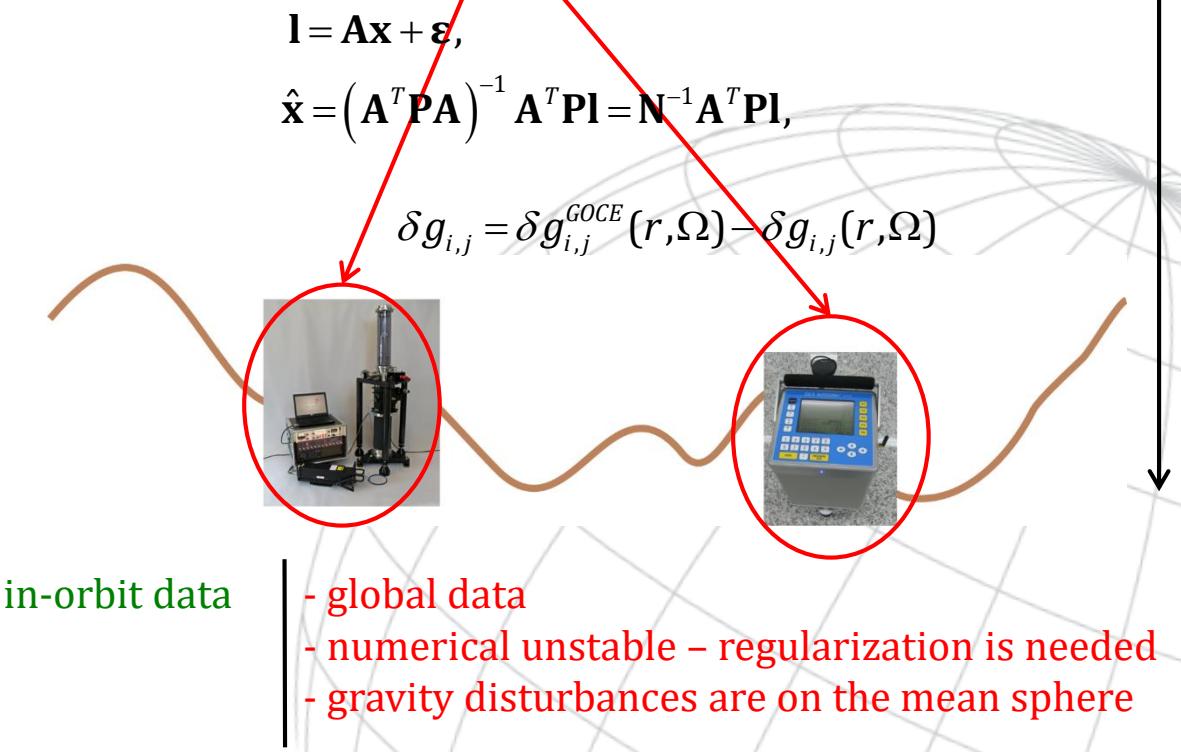


- simple method

- global data
- data are on the Earth surface

Downward continuation:

$$T_{i,j}(r, \Omega) = \frac{R}{4\pi} \int_{\Omega'} \delta g(R, \Omega') H_{i,j}(t, \psi) d\Omega', i, j = x, y, z$$

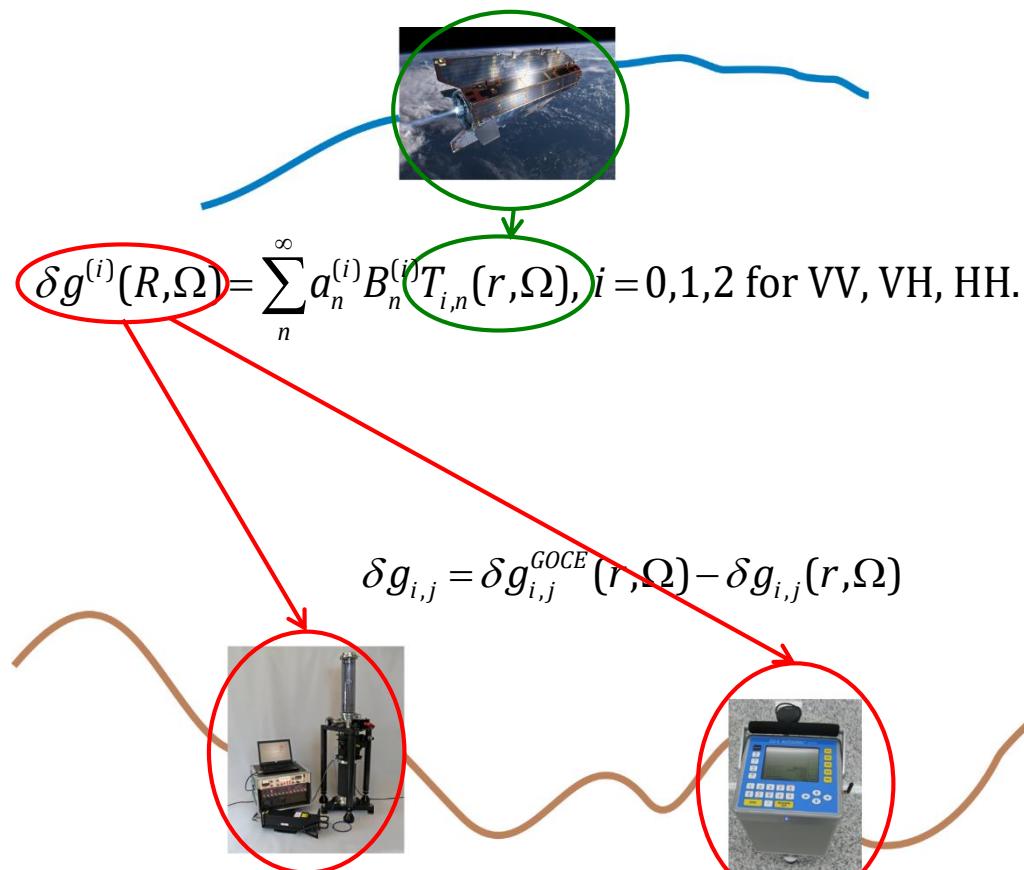


- in-orbit data

- global data
- numerical unstable – regularization is needed
- gravity disturbances are on the mean sphere

# Motivation: Approach with spectral weights

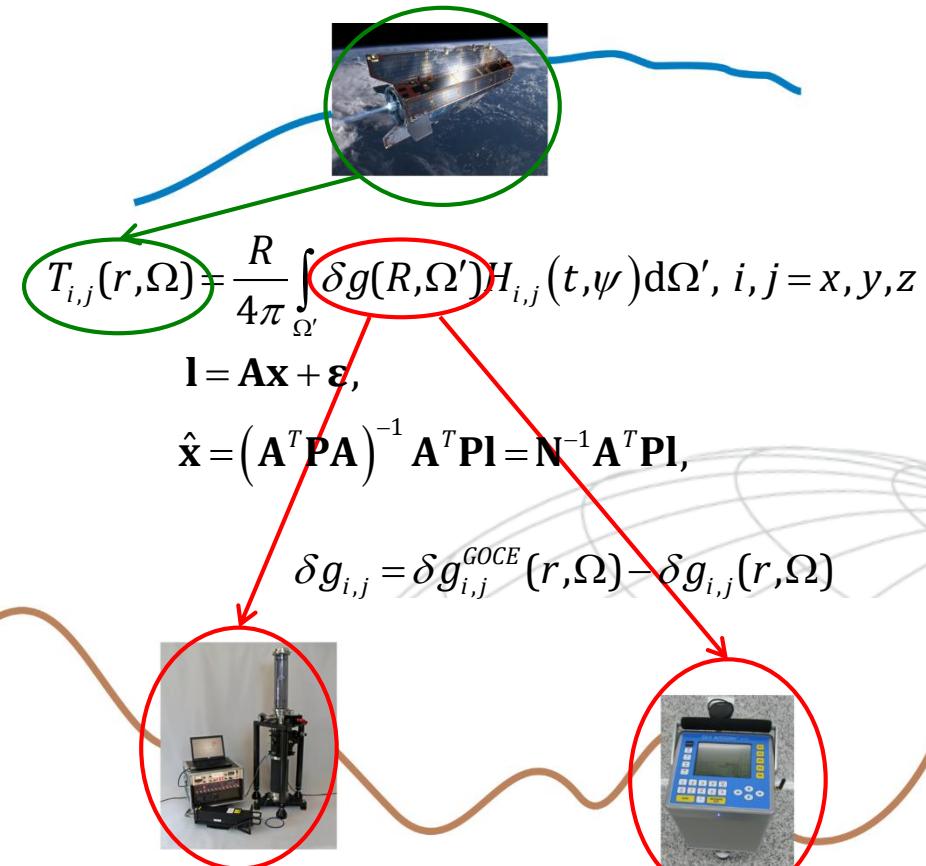
Downward continuation: application of spectral weights



- direct method
- no regularization par.
- gravity disturbances are on the Earth surface

- global data
- satellite at the mean orbital sphere

Downward continuation:



- in-orbit data

- global data
- numerical unstable – regularization is needed
- gravity disturbances are on the mean sphere

## Spectral combination method:

The solution of the spherical gradiometric boundary-value problem (BVP) for the gravity disturbances is

$$\delta g^{(VV)}(R, \Omega) = \frac{1}{4\pi} \int_{\Omega'} \sum_{n=2}^{\infty} (2n+1) B_n^{(VV)} t_n^{-1} P_{n,0}(\cos\psi) T_{zz}(r, \Omega') d\Omega',$$

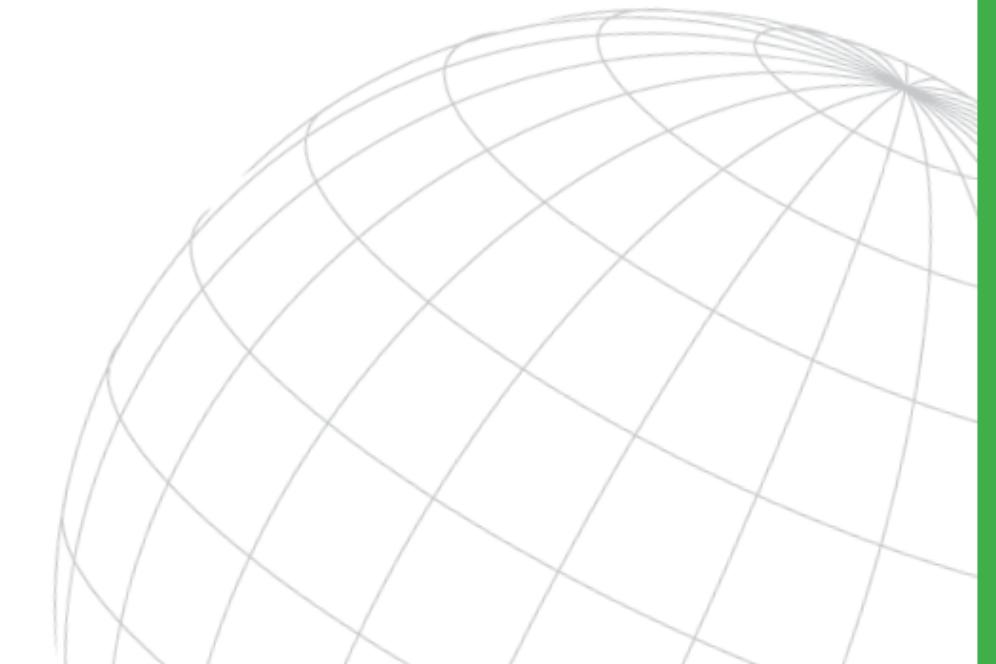
$$\delta g^{(VH)}(R, \Omega) = \frac{1}{4\pi} \int_{\Omega'} \sum_{n=2}^{\infty} (2n+1) B_n^{(VH)} t_n^{-1} P_{n,1}(\cos\psi) [T_{xz}(r, \Omega') \cos\alpha' - T_{yz}(r, \Omega') \sin\alpha'] d\Omega',$$

$$\delta g^{(HH)}(R, \Omega) = \frac{1}{4\pi} \int_{\Omega'} \sum_{n=2}^{\infty} (2n+1) B_n^{(HH)} t_n^{-1} P_{n,2}(\cos\psi) [(T_{xx}(r, \Omega') - T_{yy}(r, \Omega')) \cos 2\alpha' - 2T_{xz}(r, \Omega') \sin 2\alpha'] d\Omega',$$

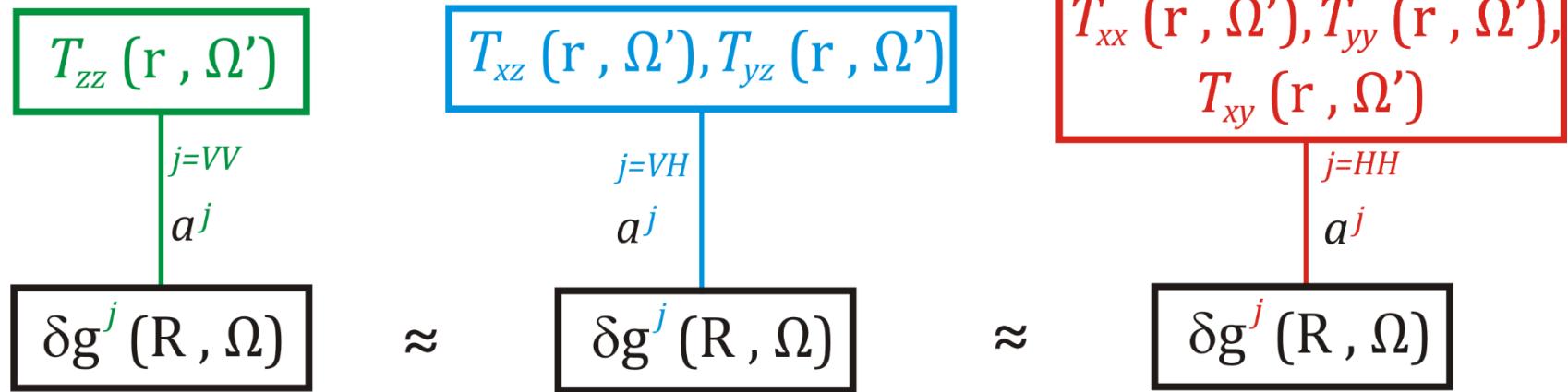
where

$$B_n^{(i)} = (-1)^{(i)} (n+1) \frac{R(n-i)!}{(n+2)!}, \quad i = 0, 1, 2 \text{ for VV, VH, HH},$$

$$t_n = (R/r)^{(n+3)}.$$



## Spectral combination method: one component estimator

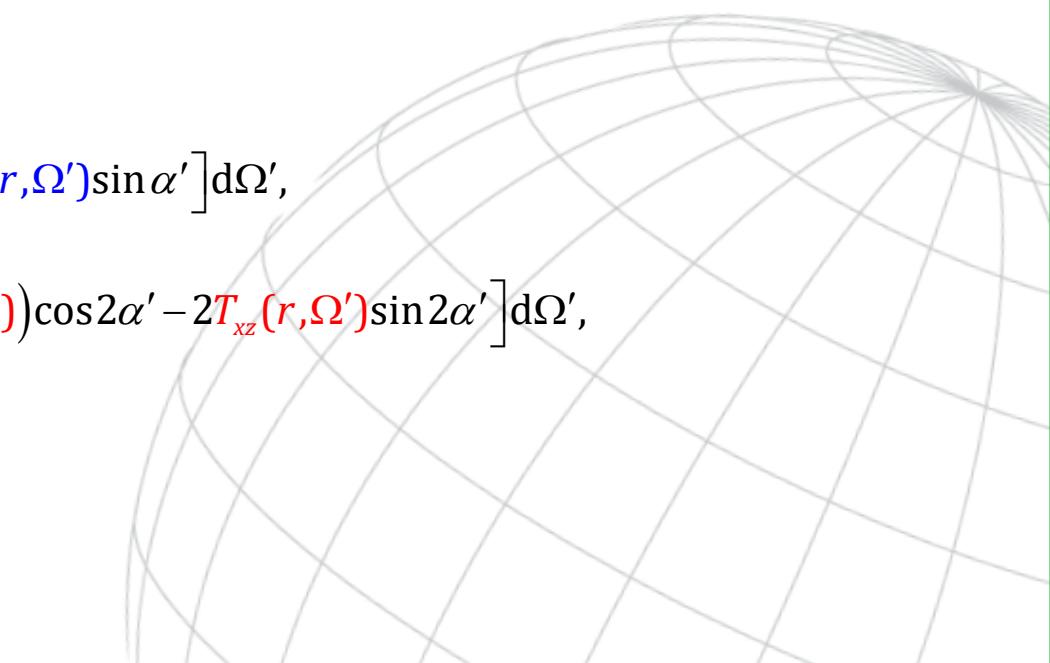


$$\delta g^{(VV)}(R, \Omega) = \frac{1}{4\pi} \int \sum_{n=2}^{\infty} a_n^{VV} (2n+1) B_n^{(VV)} P_{n,0}(\cos\psi) T_{zz}(r, \Omega') d\Omega',$$

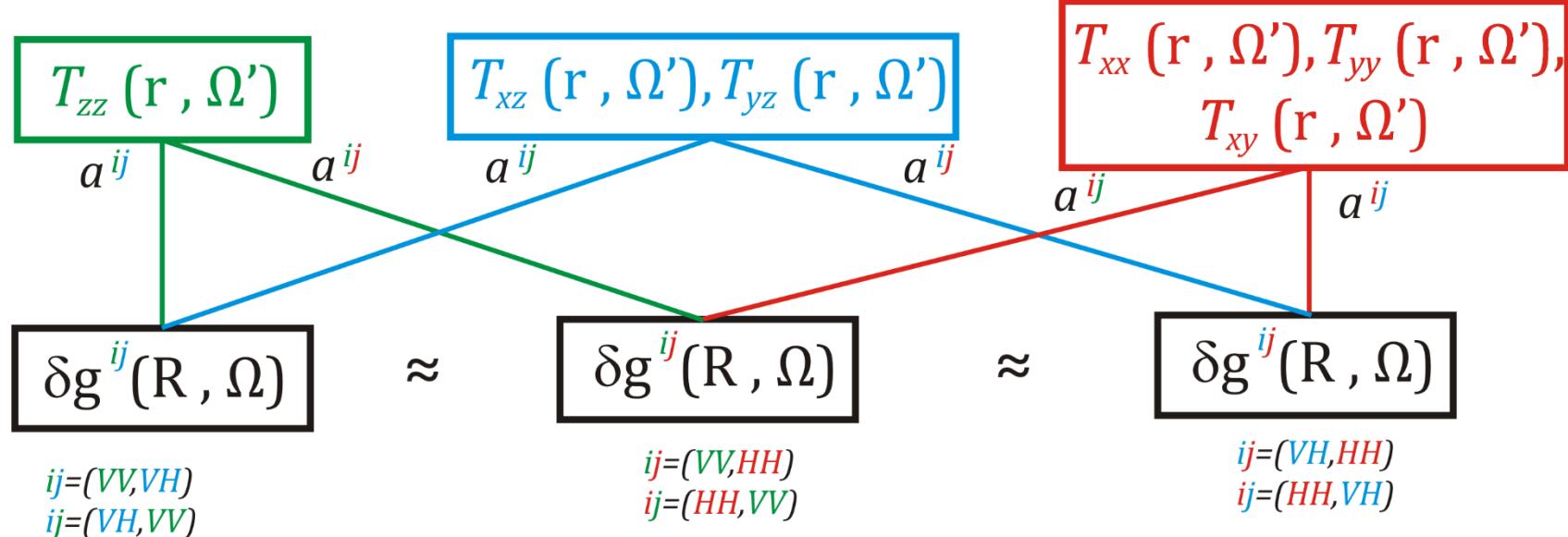
$$\delta g^{(VH)}(R, \Omega) = \frac{1}{4\pi} \int \sum_{n=2}^{\infty} a_n^{VH} (2n+1) B_n^{(VH)} P_{n,1}(\cos\psi) [T_{xz}(r, \Omega') \cos\alpha' - T_{yz}(r, \Omega') \sin\alpha'] d\Omega',$$

$$\delta g^{(HH)}(R, \Omega) = \frac{1}{4\pi} \int \sum_{n=2}^{\infty} a_n^{HH} (2n+1) B_n^{(HH)} P_{n,2}(\cos\psi) [(T_{xx}(r, \Omega') - T_{yy}(r, \Omega')) \cos 2\alpha' - 2T_{xz}(r, \Omega') \sin 2\alpha'] d\Omega',$$

$$a_n^{(i)} = \frac{t_n c_{\delta g, n}}{(t_n)^2 c_{\delta g, n} + B_n^{(i)} \sigma_{i,n}^2}$$

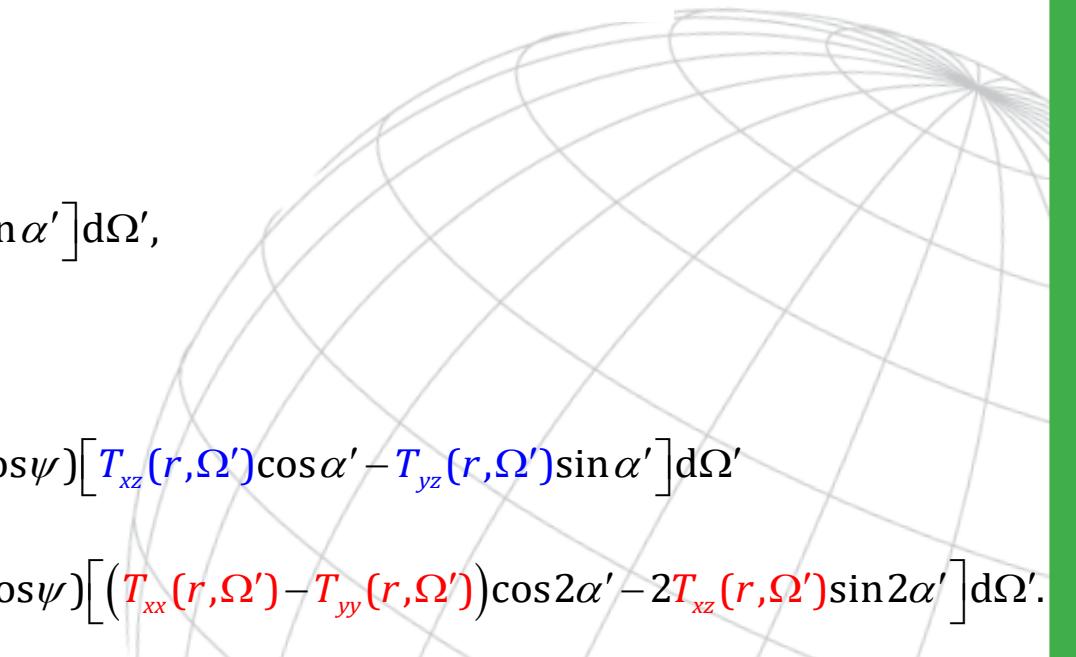


## Spectral combination method: two component estimator

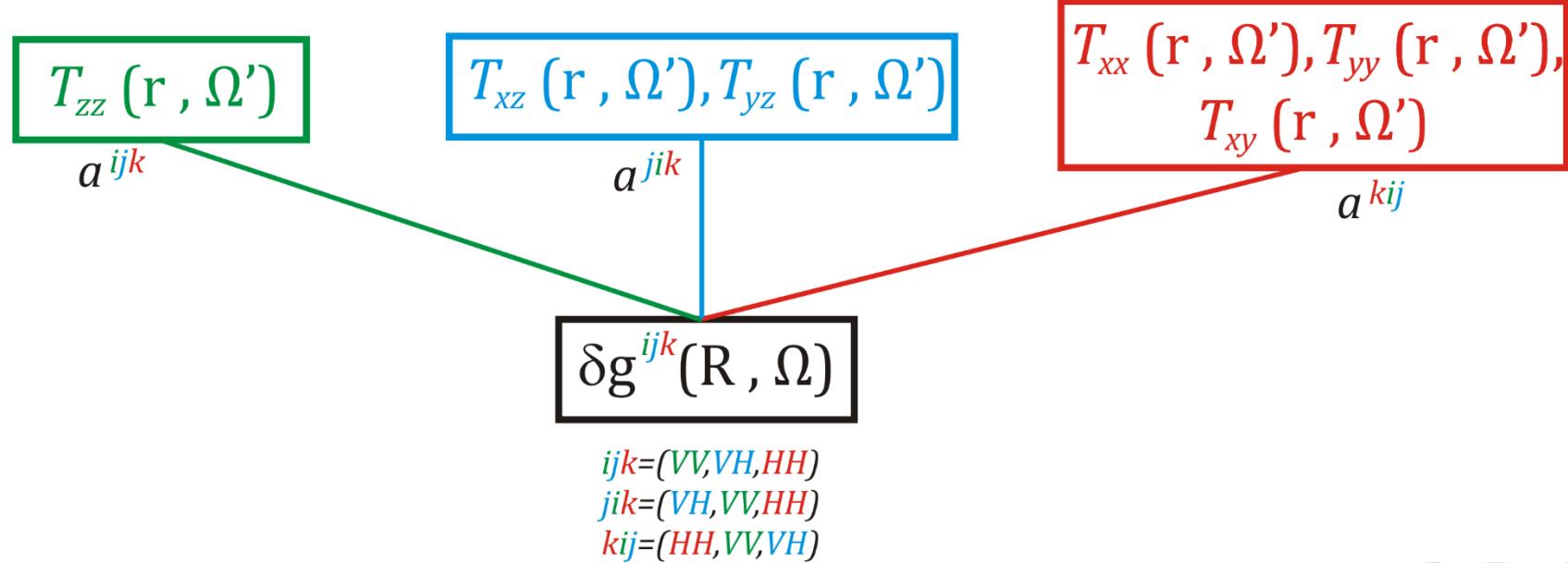


$$\begin{aligned} \delta g^{(VV,VH)}(R, \Omega) &= \frac{1}{4\pi} \int \sum_{n=2}^{\infty} a_n^{(VV,VH)} (2n+1) B_n^{(VV)} P_{n,0}(\cos\psi) T_{zz}(r, \Omega') d\Omega' \\ &+ \frac{1}{4\pi} \int \sum_{n=2}^{\infty} a_n^{(VH,VV)} (2n+1) B_n^{(VH)} P_{n,1}(\cos\psi) [T_{xz}(r, \Omega') \cos\alpha' - T_{yz}(r, \Omega') \sin\alpha'] d\Omega', \end{aligned}$$

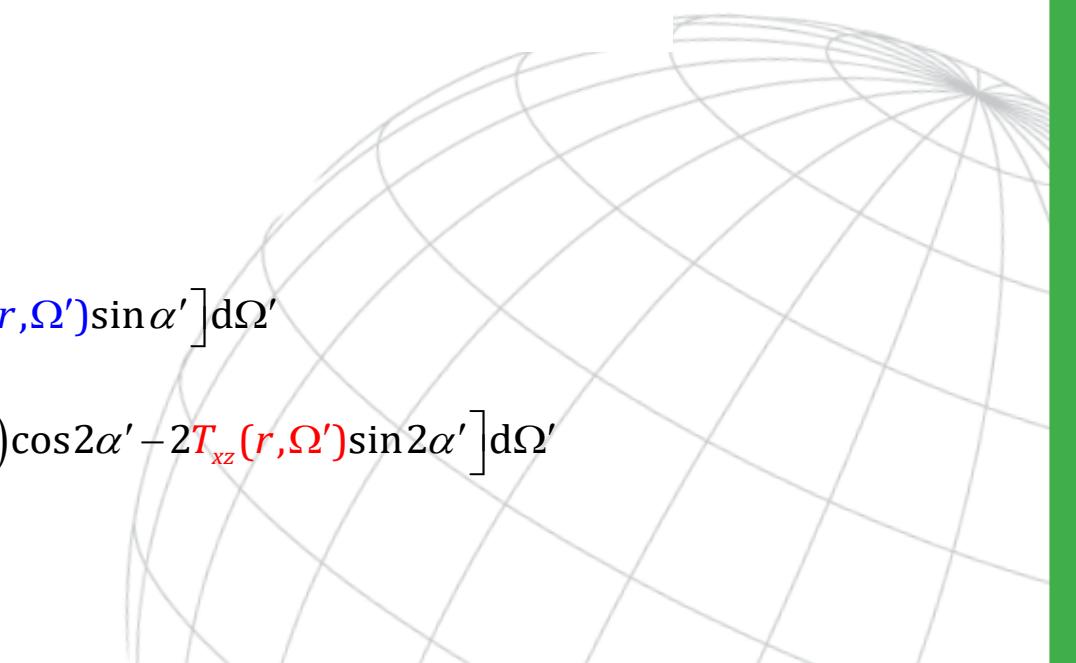
$$\begin{aligned} \delta g^{(VH,HH)}(R, \Omega) &= \frac{1}{4\pi} \int \sum_{n=2}^{\infty} a_n^{(VH,HH)} (2n+1) B_n^{(VH)} P_{n,1}(\cos\psi) [T_{xz}(r, \Omega') \cos\alpha' - T_{yz}(r, \Omega') \sin\alpha'] d\Omega' \\ &+ \frac{1}{4\pi} \int \sum_{n=2}^{\infty} a_n^{(HH,VH)} (2n+1) B_n^{(HH)} P_{n,2}(\cos\psi) [(T_{xx}(r, \Omega') - T_{yy}(r, \Omega')) \cos 2\alpha' - 2T_{xz}(r, \Omega') \sin 2\alpha'] d\Omega'. \end{aligned}$$



## Spectral combination method: three component estimator



$$\begin{aligned} \delta g^{(VV,VH,HH)}(R, \Omega) = & \frac{1}{4\pi} \int \sum_{\Omega'}^{\infty} a_n^{(VV,VH,HH)} (2n+1) B_n^{(VV)} P_{n,0}(\cos\psi) T_{zz}(r, \Omega') d\Omega' \\ & + \frac{1}{4\pi} \int \sum_{\Omega'}^{\infty} a_n^{(VH,VV,HH)} (2n+1) B_n^{(VH)} P_{n,1}(\cos\psi) [T_{xz}(r, \Omega') \cos\alpha' - T_{yz}(r, \Omega') \sin\alpha'] d\Omega' \\ & + \frac{1}{4\pi} \int \sum_{\Omega'}^{\infty} a_n^{(HH,VV,VH)} (2n+1) B_n^{(HH)} P_{n,2}(\cos\psi) [(T_{xx}(r, \Omega') - T_{yy}(r, \Omega')) \cos 2\alpha' - 2T_{xz}(r, \Omega') \sin 2\alpha'] d\Omega' \end{aligned}$$



## Closed-loop test of the method:

- Test area: Czech republic and Slovakia
- Input data: topographic heights from Earth2014, gravity data from EIGEN6C4

Coordinates of the test points.

	$\varphi$ [°]	$\lambda$ [°]	r [m]	H [m]
A	48.966667	20.116667	6363203.060	2121.332
B	48.433333	21.833333	6356990.306	92.490

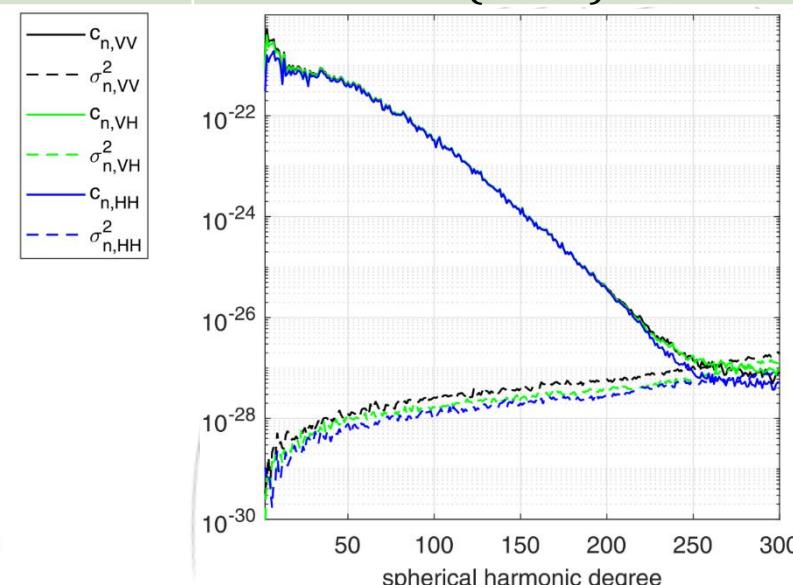
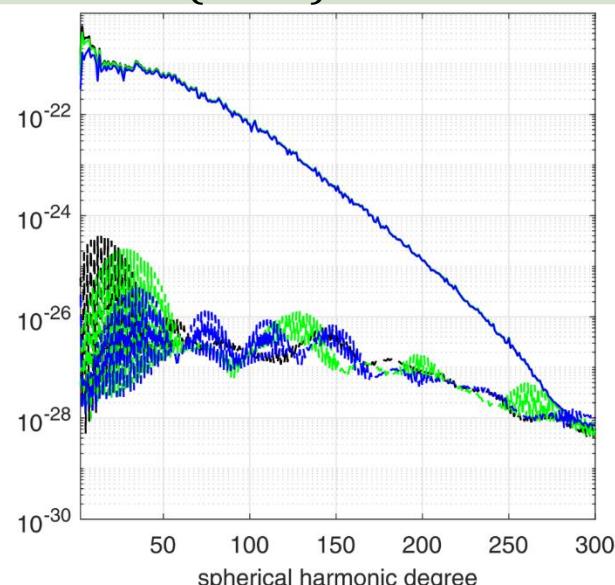
Differences (in mGal) between the estimated values of gravity disturbances and their counterparts from GGM EIGEN-6C4 up to the degree 200 (upper), 250 (middle), 300 (bottom).

	VV	VH	HH	(VV,VH)	(VV,HH)	(VH,HH)	(VV,VH,HH)
A	-0.005	-0.005	0.009	-0.005	0.009	0.009	0.009
B	-0.010	-0.008	-0.009	-0.008	-0.009	-0.009	-0.009
A	0.080	0.072	0.019	0.072	0.019	0.019	0.019
B	-0.041	-0.061	0.031	-0.061	0.031	0.031	0.031
A	-0.195	-0.067	-0.476	-0.067	-0.476	-0.476	-0.476
B	0.454	0.232	0.488	0.232	0.488	0.488	0.488



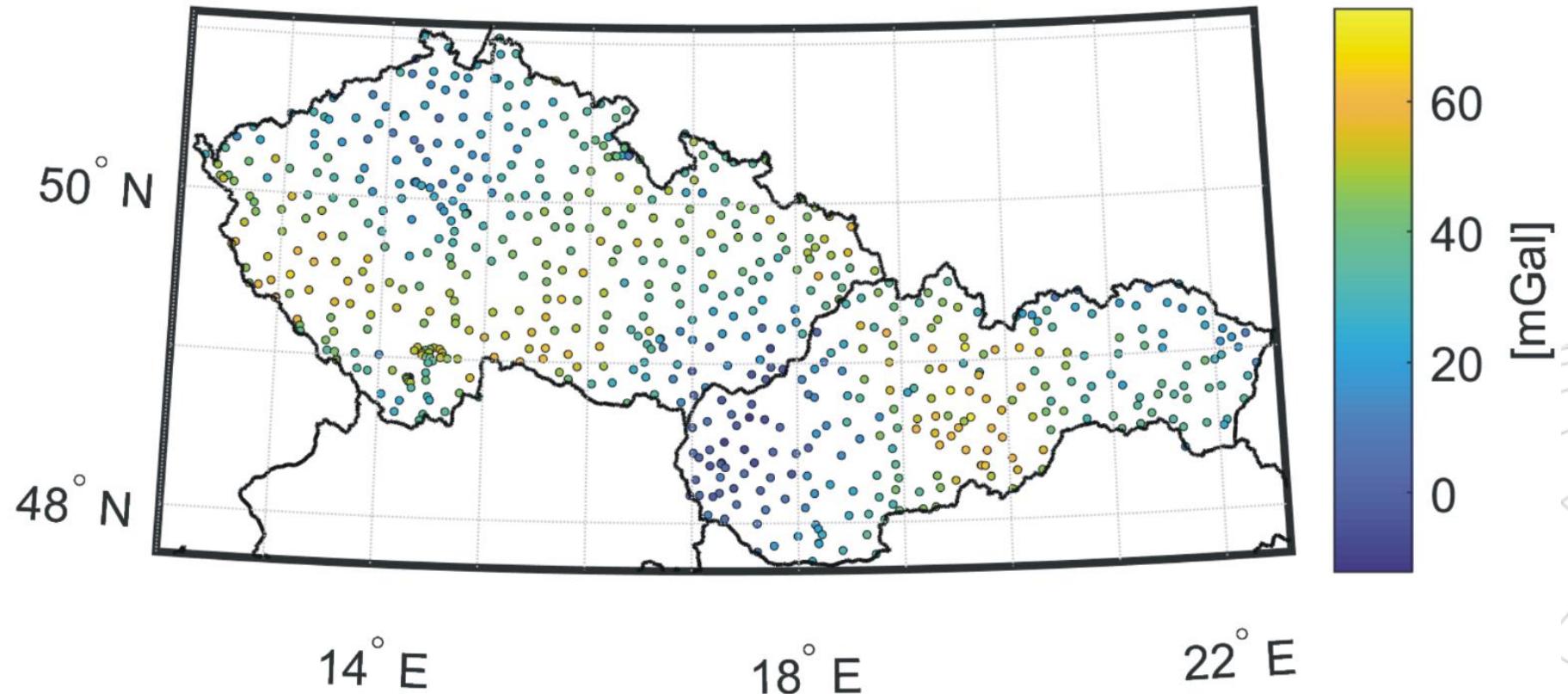
## Numerical experiment: Input data (Grids of gravitational gradients):

	Space-wise GOCE grids	GOCE+ Geoexplore II
Inputs	<ul style="list-style-type: none"> <li>- EGG_NOM_2</li> <li>- EGG_IAQ_1b</li> <li>- SST_PSO_2</li> </ul>	<ul style="list-style-type: none"> <li>- GOCE L2 TRF data</li> </ul>
Data period	1 <sup>st</sup> November 2009 – 20 <sup>th</sup> October 2013	November 2009 – October 2013
Grid area	$\varphi \in [89.9^\circ; -89.9^\circ]; \lambda \in [-179.9^\circ; 179.9^\circ]$	$\varphi \in [83^\circ; -83^\circ]; \lambda \in [-180^\circ; 180^\circ]$
Grid resolution	0.2°	1/6°
Reference radius	6600000 m	6621830 m
Outputs	$V_{nn}, V_{ee}, V_{rr}, V_{en}, V_{er}, V_{nr}$	$T_{xx}, T_{xy}, T_{xz}, T_{yy}, T_{yz}, T_{zz}$
Reference	Gatti et al. (2014)	Sebera et al. (2014)

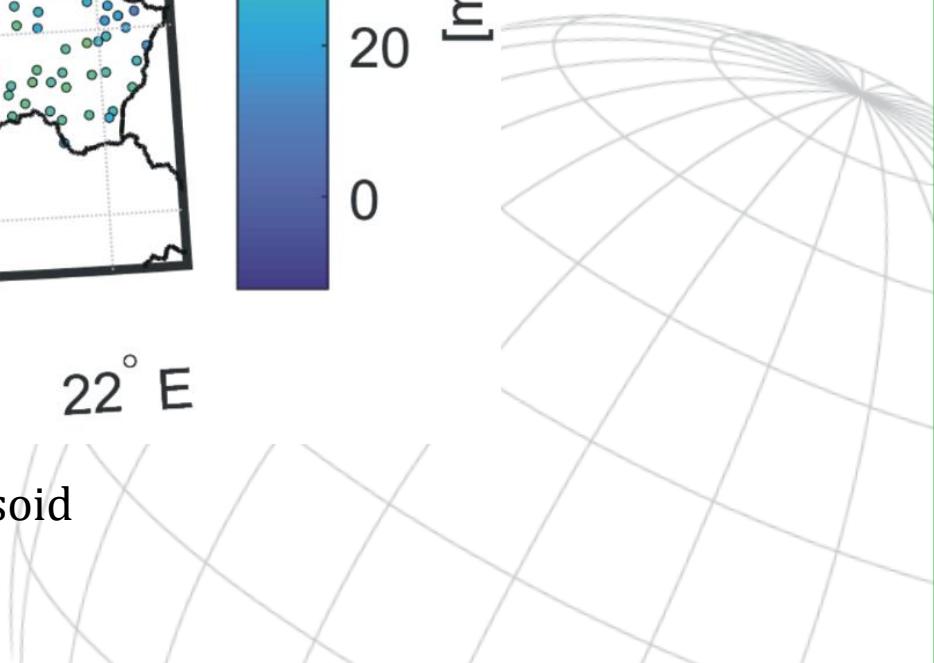


## Numerical experiment: Input data (terrestrial data):

- The terrestrial data over Czech republic (421 points of Czech gravimetric network)
- The gravity accelerations on 233 points of Slovak gravimetric network



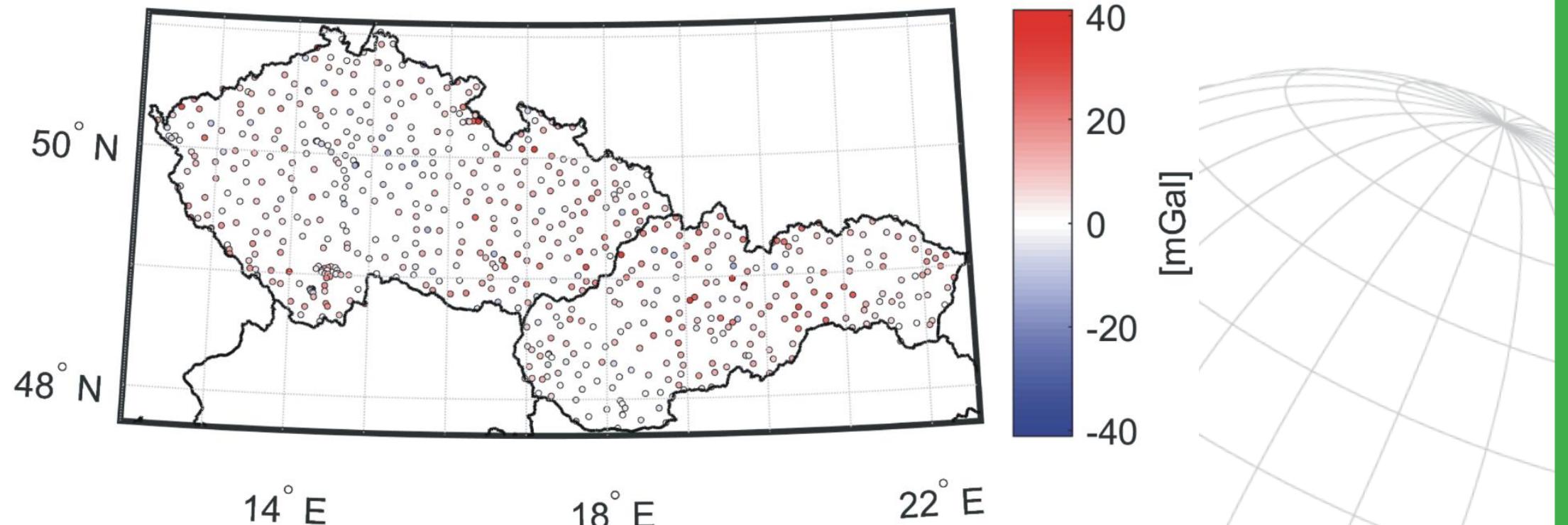
- The normal gravity was evaluated with respect to GRS80 reference ellipsoid



**Results:**  $\Delta\delta g = \delta g^{ter} - (\delta g_{SDWC}^{ter} + \delta g_{201-2190}^{\text{EGM2008}} + \delta g_{2191-5480}^{\text{dV\_ELL\_Earth2014\_5480}})$

Differences (in mGal) between terrestrial gravity anomalies and gravity anomalies obtained from spectral combination method + EGM2008 + RTM (GOCE+ Geoexplore II grids).

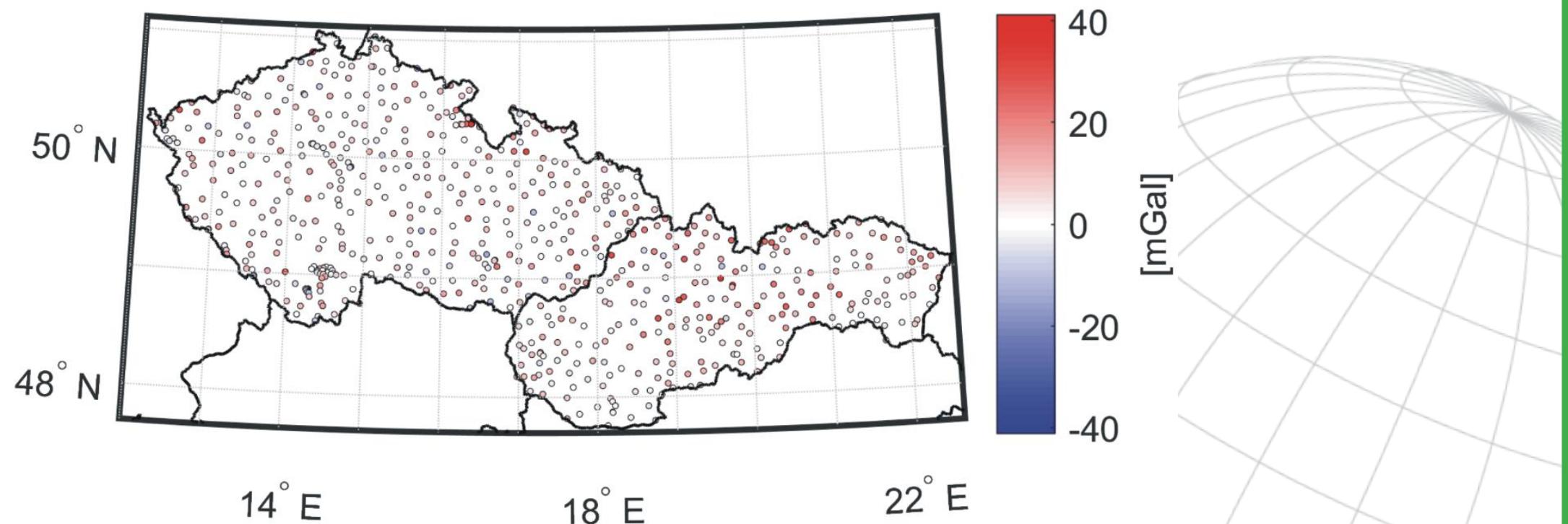
	VV	VH	HH	(VV,VH)	(VV,HH)	(VH,HH)	(VV,VH,HH)
std	8.284	8.108	8.114	8.108	8.114	8.114	8.114
max	39.841	38.907	39.033	38.904	39.050	39.041	39.042
min	-14.762	-14.495	-13.815	-14.498	-13.799	-13.808	-13.807
mean	6.252	6.332	6.446	6.329	6.462	6.453	6.454



**Results:**  $\Delta\delta g = \delta g^{ter} - (\delta g_{SDWC}^{ter} + \delta g_{201-2190}^{EGM2008} + \delta g_{2191-5480}^{dV\_ELL\_Earth2014\_5480})$

Differences (in mGal) between terrestrial gravity anomalies and gravity anomalies obtained from spectral combination method + EGM2008 + RTM (Space-wise GOCE grids).

	VV	VH	HH	(VV,VH)	(VV,HH)	(VH,HH)	(VV,VH,HH)
<b>std</b>	7.873	7.885	7.873	7.885	7.873	7.873	7.873
<b>max</b>	40.929	41.142	41.048	41.142	41.048	41.049	41.049
<b>min</b>	-17.612	-17.624	-17.781	-17.624	-17.780	-17.779	-17.779
<b>mean</b>	6.294	6.312	6.295	6.312	6.295	6.296	6.296



## Conclusion:

- we presented an alternative method for validation of GOCE-based gravitational gradients grids based on the spectral combination method,
- we validated two sets of grids, namely Space-wise GOCE grids and grids from the GOCE+Geoexplore II project,
- slightly better fit with respect to terrestrial data was achieved from Space-wise GOCE grids.



Thank you for your attention  
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