VALIDATION OF GOCE-BASED GRAVITATIONAL GRADIENTS GRIDS BY SPECTRAL COMBINATION METHOD

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Content:

- Motivation,
- Spectral combination method,
- Numerical experiment,
- Results,
- Conclusion,

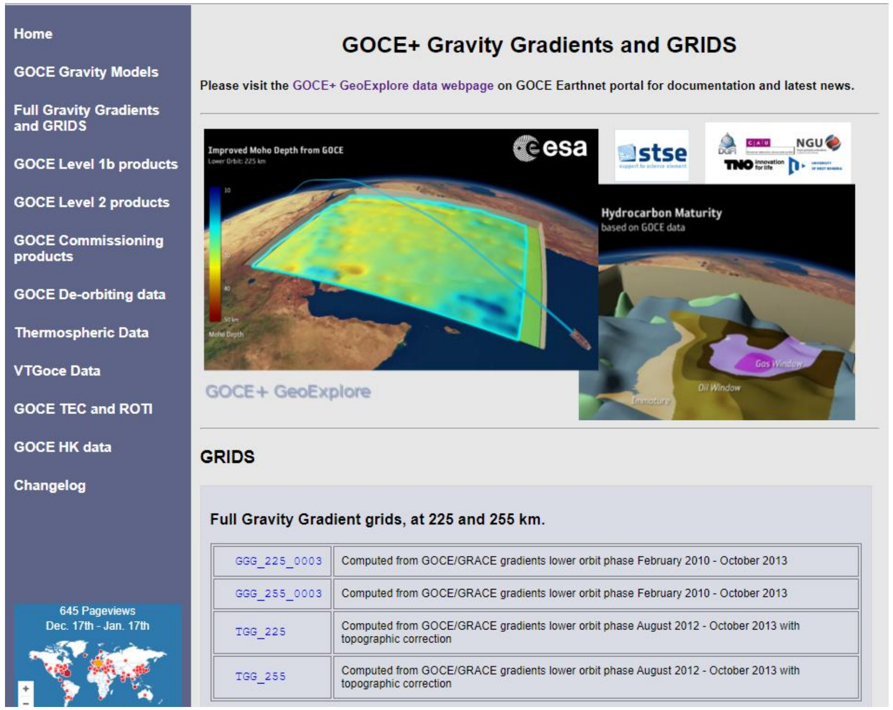
Motivation:

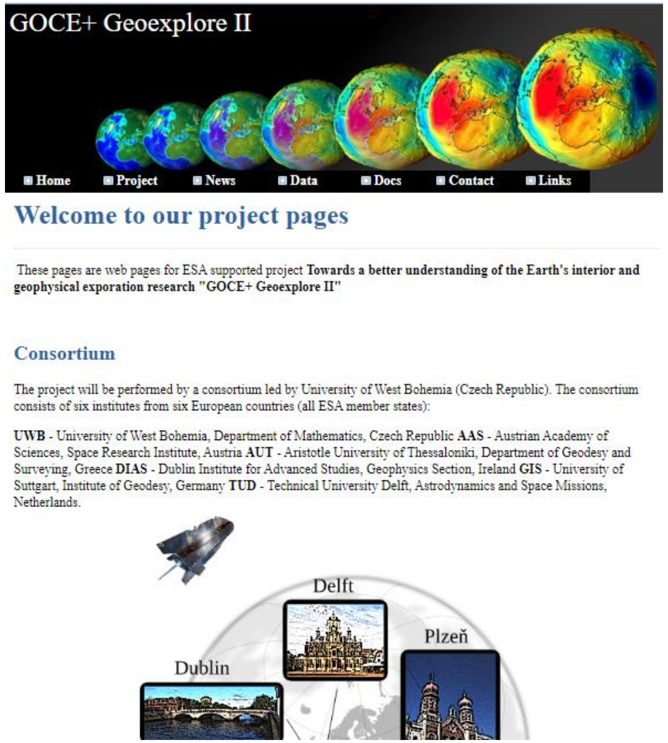
- grids of gravitational gradients produced by three groups
  (see, ESA-funded GOCE+ GeoExplore project, GOCE+ GeoExplore II project or Space-wise GOCE products),

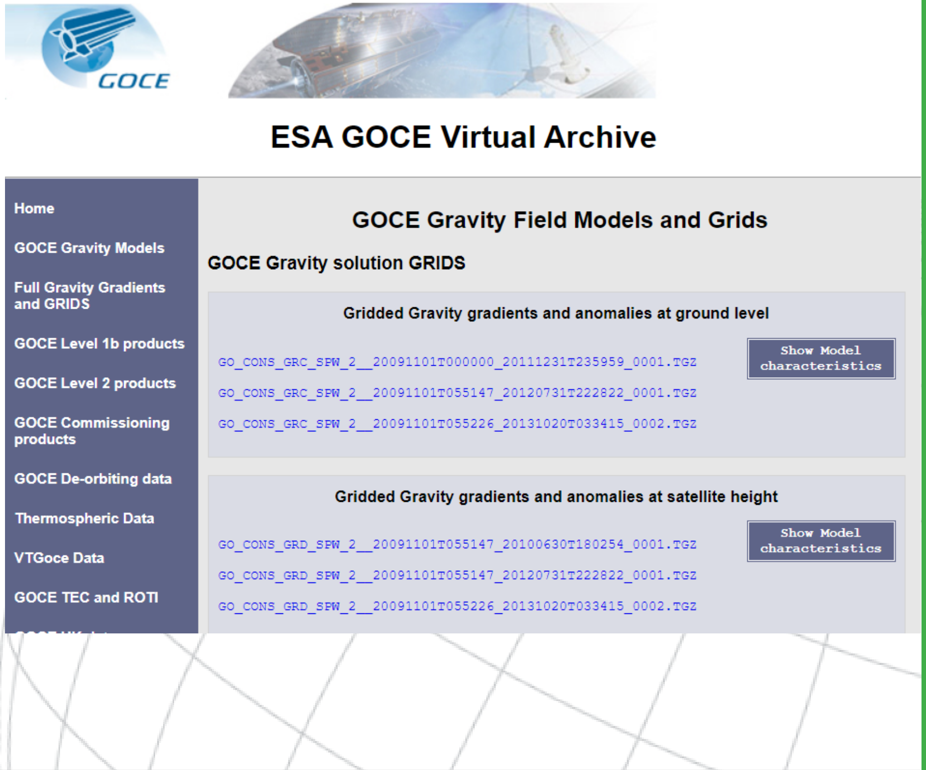
- a method for recovering gravitational field quantities on the Earth surface,

- validation of GOCE-based gravitational grids.

ESA GOCE Virtual Archive



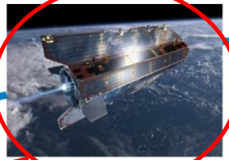




Motivation: Classical approach to validation

Upward continuation:

$$dT_{i,j} = T_{i,j}^{GOCE}(r,\Omega) - T_{i,j}(r,\Omega)$$

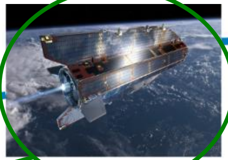


$$T_{i,j}(r,\Omega) = \frac{R}{4\pi}\int_{\Omega'} \delta g(R,\Omega') H_{i,j}(t,\psi)\,\mathrm{d}\Omega',\; i,j = x,y,z$$

- simple method

- global data
- data are on the Earth surface

Downward continuation:



$$T_{i,j}(r,\Omega) = \frac{R}{4\pi}\int_{\Omega'} \delta g(R,\Omega') H_{i,j}(t,\psi)\,\mathrm{d}\Omega',\; i,j = x,y,z$$

$$\mathbf{l} = \mathbf{A}\mathbf{x} + \boldsymbol{\varepsilon},$$

$$\hat{\mathbf{x}} = \left(\mathbf{A}^T\mathbf{P}\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{P}\mathbf{l} = \mathbf{N}^{-1}\mathbf{A}^T\mathbf{P}\mathbf{l},$$

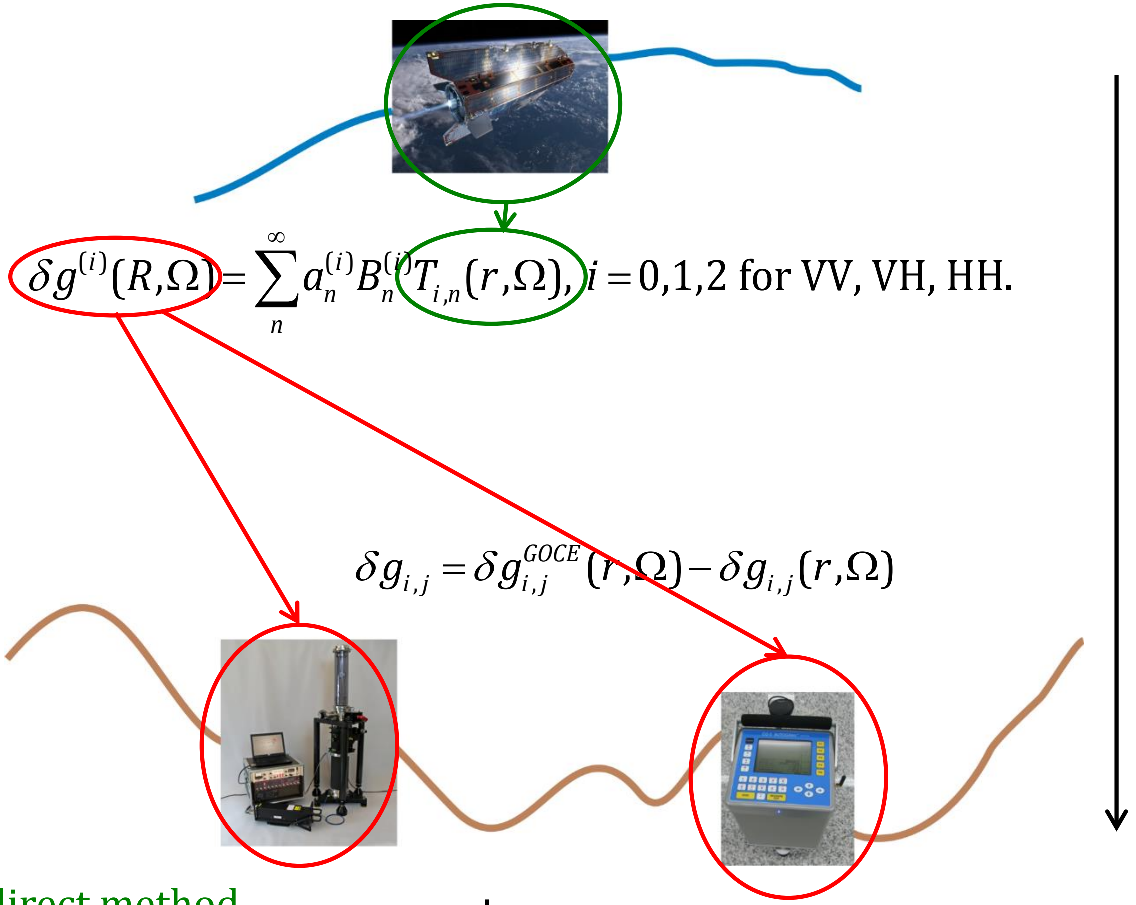
$$\delta g_{i,j} = \delta g_{i,j}^{GOCE}(r,\Omega) - \delta g_{i,j}(r,\Omega)$$

- in-orbit data

- global data
- numerical unstable – regularization is needed
- gravity disturbances are on the mean sphere

Motivation: Approach with spectral weigths

Downward continuation: application of spectral weights



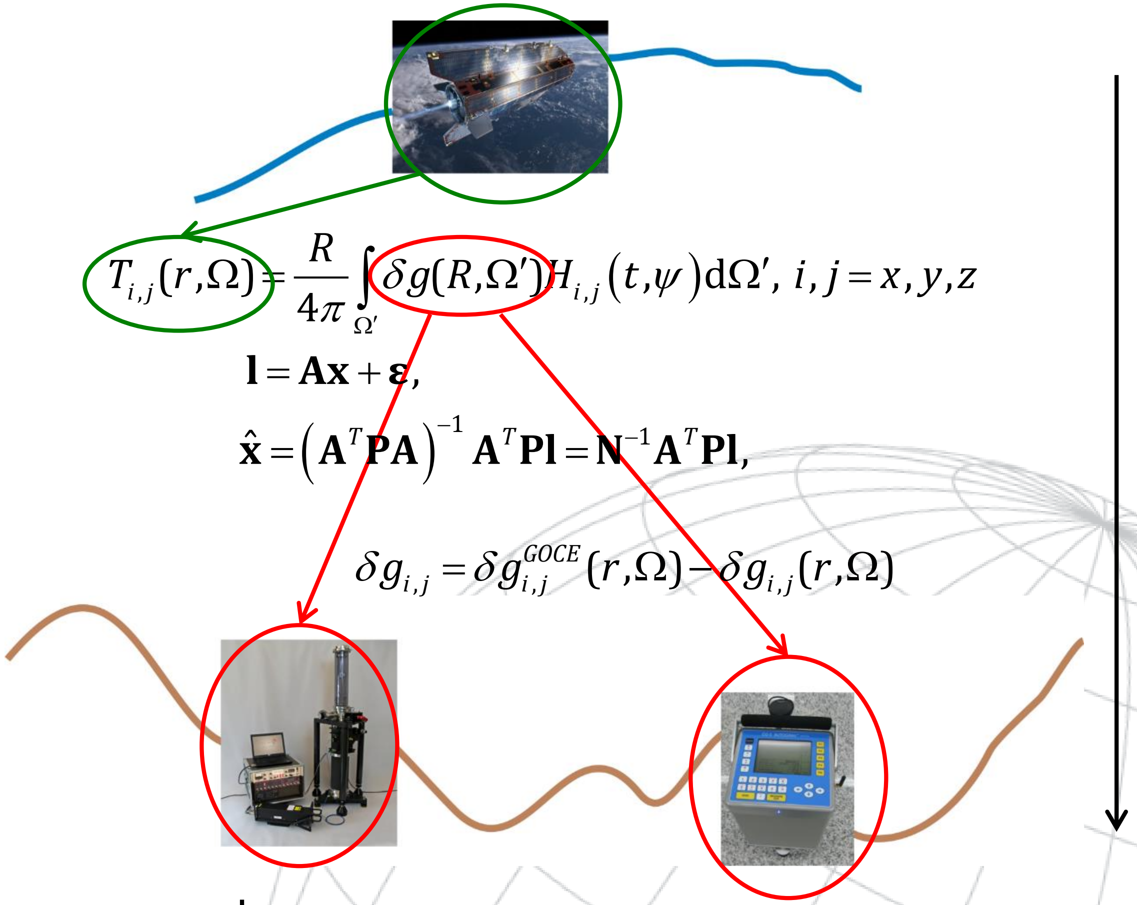
$$\delta g^{(i)}(R,\Omega) = \sum_{n}^{\infty} a_n^{(i)} B_n^{(i)} T_{i,n}(r,\Omega),\ i = 0,1,2 \text{ for VV, VH, HH.}$$

$$\delta g_{i,j} = \delta g_{i,j}^{GOCE}(r,\Omega) - \delta g_{i,j}(r,\Omega)$$

- direct method
- no regularization par.
- gravity disturbances are on the Earth surface

- global data
- satellite at the mean orbital sphere

Downward continuation:



$$T_{i,j}(r,\Omega) = \frac{R}{4\pi} \int_{\Omega'} \delta g(R,\Omega') H_{i,j}(t,\psi)\, \mathrm{d}\Omega',\ i,j = x,y,z$$

$$\mathbf{l} = \mathbf{A}\mathbf{x} + \boldsymbol{\varepsilon},$$

$$\hat{\mathbf{x}} = \left(\mathbf{A}^T \mathbf{P} \mathbf{A}\right)^{-1} \mathbf{A}^T \mathbf{P} \mathbf{l} = \mathbf{N}^{-1} \mathbf{A}^T \mathbf{P} \mathbf{l},$$

$$\delta g_{i,j} = \delta g_{i,j}^{GOCE}(r,\Omega) - \delta g_{i,j}(r,\Omega)$$

- in-orbit data

- global data
- numerical unstable – regularization is needed
- gravity disturbances are on the mean sphere

Spectral combination method:

The solution of the spherical gradiometric boundary-value problem (BVP) for the gravity disturbances is

$$\delta g^{(VV)}(R,\Omega)=\frac{1}{4\pi}\int_{\Omega'}\sum_{n=2}^{\infty}(2n+1)B_n^{(VV)}t_n^{-1}P_{n,0}(\cos\psi)T_{zz}(r,\Omega')\mathrm{d}\Omega',$$

$$\delta g^{(VH)}(R,\Omega)=\frac{1}{4\pi}\int_{\Omega'}\sum_{n=2}^{\infty}(2n+1)B_n^{(VH)}t_n^{-1}P_{n,1}(\cos\psi)\left[T_{xz}(r,\Omega')\cos\alpha'-T_{yz}(r,\Omega')\sin\alpha'\right]\mathrm{d}\Omega',$$

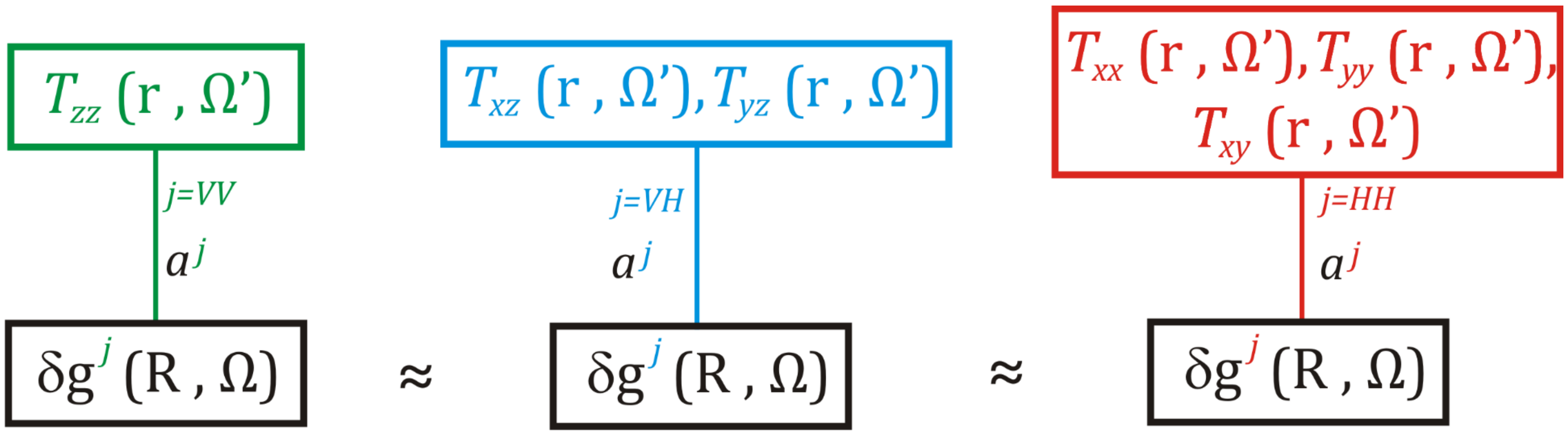
$$\delta g^{(HH)}(R,\Omega)=\frac{1}{4\pi}\int_{\Omega'}\sum_{n=2}^{\infty}(2n+1)B_n^{(HH)}t_n^{-1}P_{n,2}(\cos\psi)\left[\left(T_{xx}(r,\Omega')-T_{yy}(r,\Omega')\right)\cos 2\alpha'-2T_{xz}(r,\Omega')\sin 2\alpha'\right]\mathrm{d}\Omega',$$

where

$$B_n^{(i)}=(-1)^{(i)}(n+1)\frac{R(n-i)!}{(n+2)!},\ i=0,1,2 \text{ for VV, VH, HH,}$$

$$t_n=(R/r)^{(n+3)}.$$

Spectral combination method: one component estimator



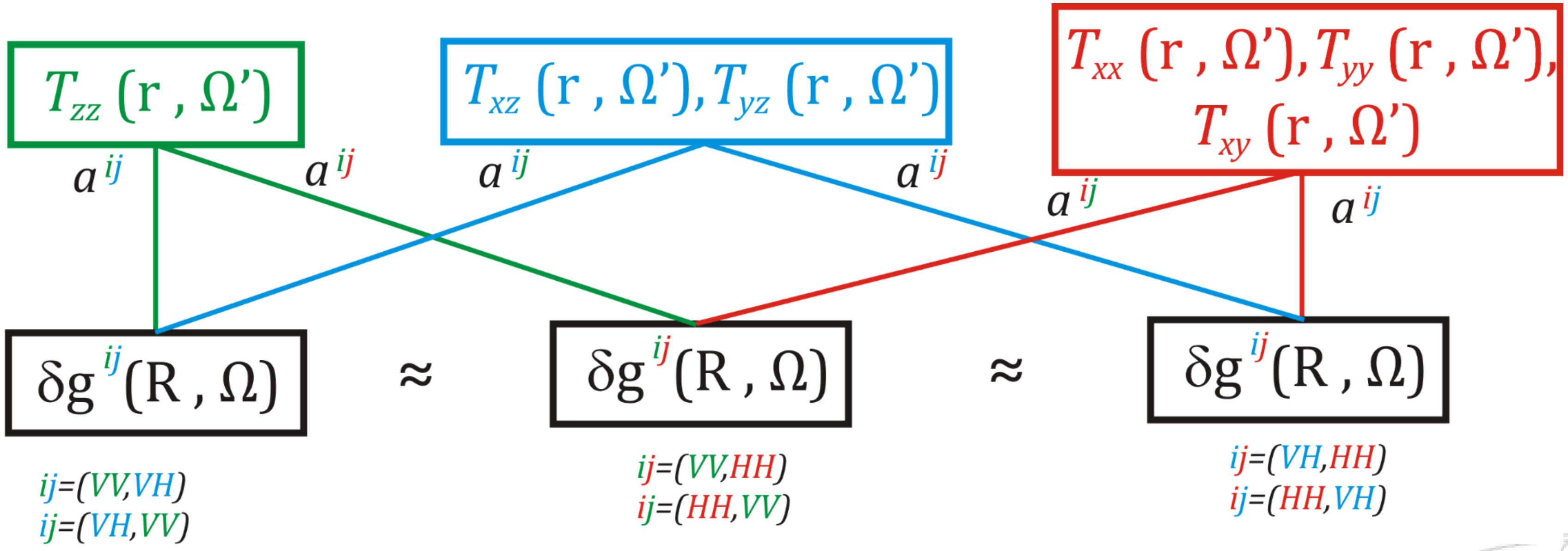
$$\delta g^{(VV)}(R,\Omega)=\frac{1}{4\pi}\int_{\Omega'}\sum_{n=2}^{\infty}a_n^{VV}(2n+1)B_n^{(VV)}P_{n,0}(\cos\psi)T_{zz}(r,\Omega')\mathrm{d}\Omega',$$

$$\delta g^{(VH)}(R,\Omega)=\frac{1}{4\pi}\int_{\Omega'}\sum_{n=2}^{\infty}a_n^{VH}(2n+1)B_n^{(VH)}P_{n,1}(\cos\psi)\left[T_{xz}(r,\Omega')\cos\alpha'-T_{yz}(r,\Omega')\sin\alpha'\right]\mathrm{d}\Omega',$$

$$\delta g^{(HH)}(R,\Omega)=\frac{1}{4\pi}\int_{\Omega'}\sum_{n=2}^{\infty}a_n^{HH}(2n+1)B_n^{(HH)}P_{n,2}(\cos\psi)\left[\left(T_{xx}(r,\Omega')-T_{yy}(r,\Omega')\right)\cos 2\alpha'-2T_{xz}(r,\Omega')\sin 2\alpha'\right]\mathrm{d}\Omega',$$

$$a_n^{(i)}=\frac{t_n c_{\delta g,n}}{(t_n)^2 c_{\delta g,n}+B_n^{(i)}\sigma_{i,n}^2}$$

Spectral combination method: two component estinmator



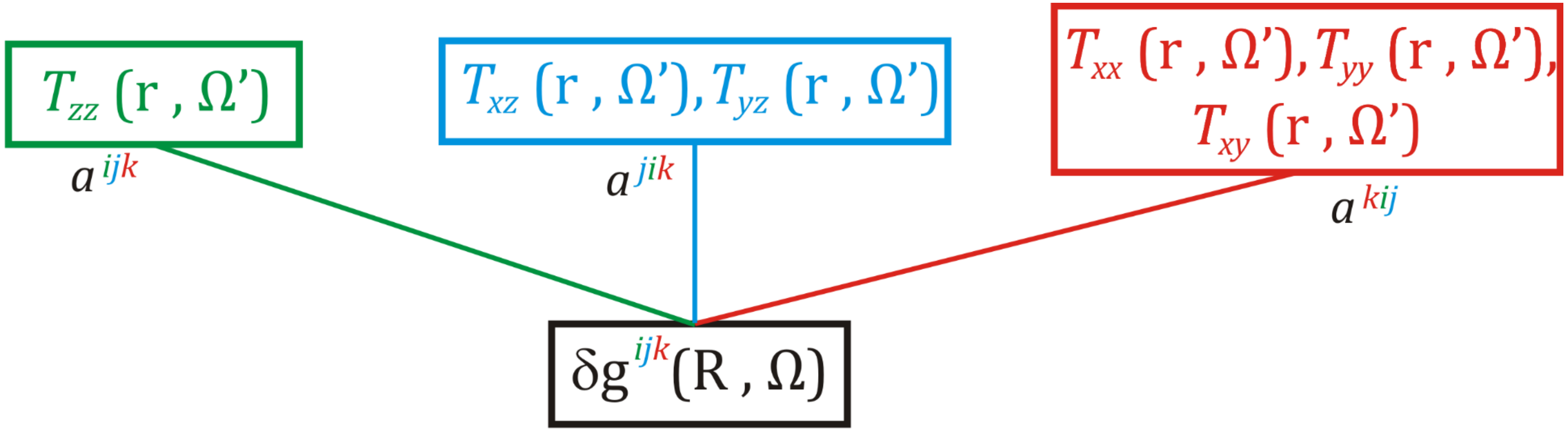
$$\delta g^{(VV,VH)}(R,\Omega) = \frac{1}{4\pi}\int_{\Omega'}\sum_{n=2}^{\infty} a_n^{(VV,VH)}(2n+1)B_n^{(VV)}P_{n,0}(\cos\psi)T_{zz}(r,\Omega')\mathrm{d}\Omega'$$

$$+\frac{1}{4\pi}\int_{\Omega'}\sum_{n=2}^{\infty} a_n^{(VH,VV)}(2n+1)B_n^{(VH)}P_{n,1}(\cos\psi)\left[T_{xz}(r,\Omega')\cos\alpha' - T_{yz}(r,\Omega')\sin\alpha'\right]\mathrm{d}\Omega',$$

$$\delta g^{(VH,HH)}(R,\Omega) = \frac{1}{4\pi}\int_{\Omega'}\sum_{n=2}^{\infty} a_n^{(VH,HH)}(2n+1)B_n^{(VH)}P_{n,1}(\cos\psi)\left[T_{xz}(r,\Omega')\cos\alpha' - T_{yz}(r,\Omega')\sin\alpha'\right]\mathrm{d}\Omega'$$

$$+\frac{1}{4\pi}\int_{\Omega'}\sum_{n=2}^{\infty} a_n^{(HH,VH)}(2n+1)B_n^{(HH)}P_{n,2}(\cos\psi)\left[\left(T_{xx}(r,\Omega') - T_{yy}(r,\Omega')\right)\cos 2\alpha' - 2T_{xz}(r,\Omega')\sin 2\alpha'\right]\mathrm{d}\Omega'.$$

Spectral combination method: three component estimator



ijk=(VV,VH,HH)
jik=(VH,VV,HH)
kij=(HH,VV,VH)

$$\delta g^{(VV,VH,HH)}(R,\Omega) = \frac{1}{4\pi}\int_{\Omega'}\sum_{n=2}^{\infty} a_n^{(VV,VH,HH)}(2n+1)B_n^{(VV)}P_{n,0}(\cos\psi)T_{zz}(r,\Omega')\,d\Omega'$$
$$+\frac{1}{4\pi}\int_{\Omega'}\sum_{n=2}^{\infty} a_n^{(VH,VV,HH)}(2n+1)B_n^{(VH)}P_{n,1}(\cos\psi)\left[T_{xz}(r,\Omega')\cos\alpha' - T_{yz}(r,\Omega')\sin\alpha'\right]d\Omega'$$
$$+\frac{1}{4\pi}\int_{\Omega'}\sum_{n=2}^{\infty} a_n^{(HH,VV,VH)}(2n+1)B_n^{(HH)}P_{n,2}(\cos\psi)\left[\left(T_{xx}(r,\Omega') - T_{yy}(r,\Omega')\right)\cos 2\alpha' - 2T_{xz}(r,\Omega')\sin 2\alpha'\right]d\Omega'$$

Closed-loop test of the method:

- Test area: Czech republic and Slovakia
- Input data: topographic heights from Earth2014, gravity data from EIGEN6C4

Coordinates of the test points.

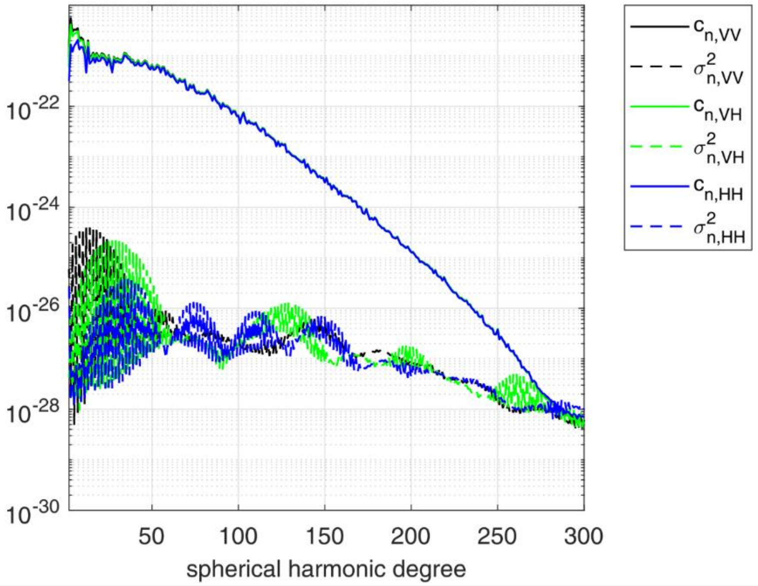
	φ [°]	λ [°]	r [m]	H [m]
A	48.966667	20.116667	6363203.060	2121.332
B	48.433333	21.833333	6356990.306	92.490

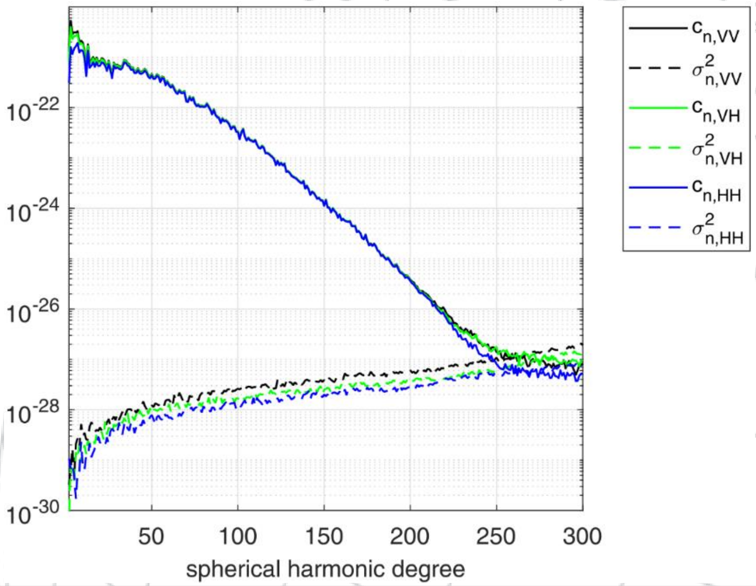
Differences (in mGal) between the estimated values of gravity disturbances and their counterparts from GGM EIGEN-6C4 up to the degree 200 (upper), 250 (middle), 300 (bottom).

	VV	VH	HH	(VV,VH)	(VV,HH)	(VH,HH)	(VV,VH,HH)
A	-0.005	-0.005	0.009	-0.005	0.009	0.009	0.009
B	-0.010	-0.008	-0.009	-0.008	-0.009	-0.009	-0.009
A	0.080	0.072	0.019	0.072	0.019	0.019	0.019
B	-0.041	-0.061	0.031	-0.061	0.031	0.031	0.031
A	-0.195	-0.067	-0.476	-0.067	-0.476	-0.476	-0.476
B	0.454	0.232	0.488	0.232	0.488	0.488	0.488

Numerical experiment: Input data (Grids of gravitational gradients):

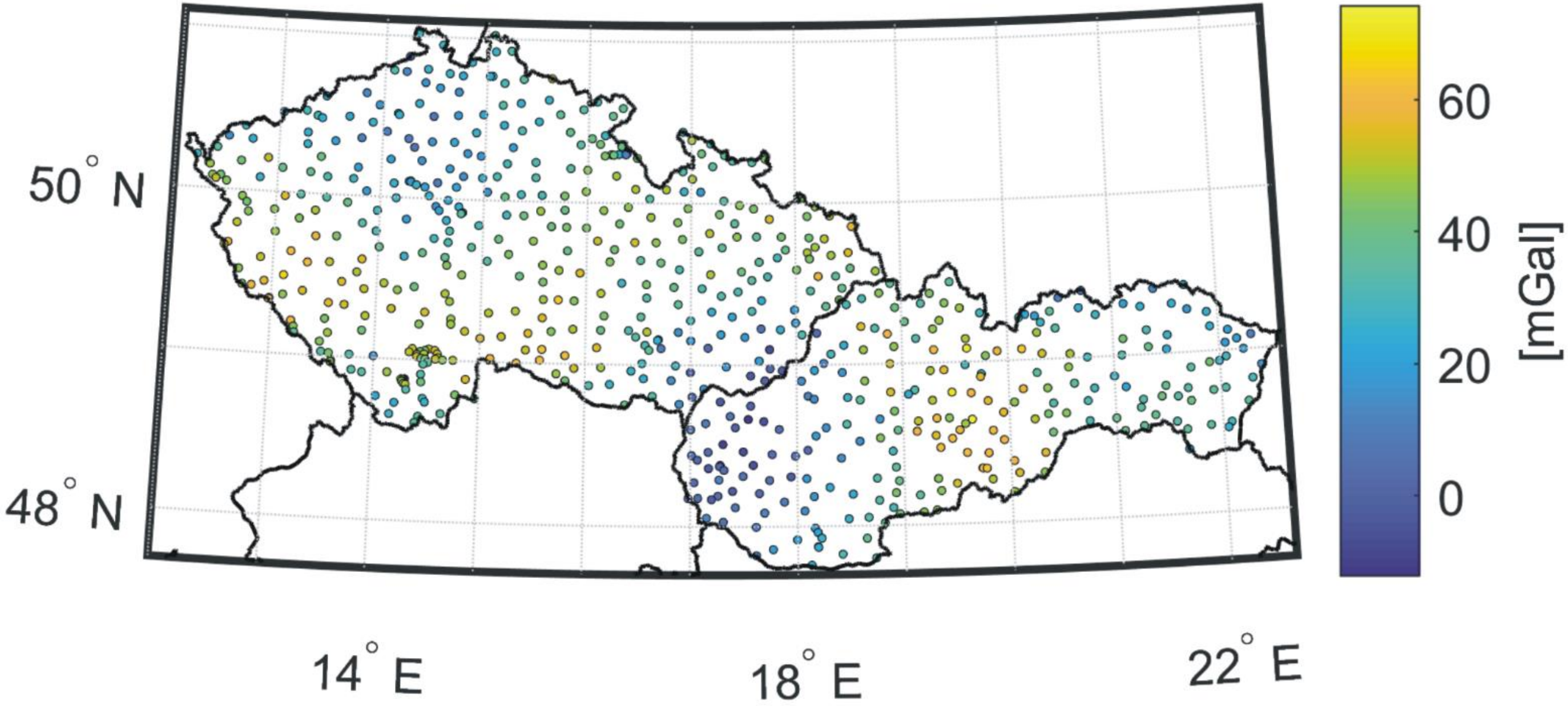
	Space-wise GOCE grids	GOCE+ Geoexplore II
Inputs	- EGG_NOM_2 - EGG_IAQ_1b - SST_PSO_2	- GOCE L2 TRF data
Data period	1st November 2009 – 20th October 2013	November 2009 – October 2013
Grid area	$\varphi \in [89.9°;-89.9°]$; $\lambda \in [-179.9°;179.9°]$	$\varphi \in [83°;-83°]$; $\lambda \in [-180°;180-1/6°]$
Grid resolution	0.2°	1/6°
Reference radius	6600000 m	6621830 m
Outputs	$V_{nn}, V_{ee}, V_{rr}, V_{en}, V_{er}, V_{nr}$	$T_{xx}, T_{xy}, T_{xz}, T_{yy}, T_{yz}, T_{zz}$
Reference	Gatti et al. (2014)	Sebera et al. (2014)





Numerical experiment: Input data (terrestrial data):

- The terrestrial data over Czech republic (421 points of Czech gravimetric network)
- The gravity accelerations on 233 points of Slovak gravimetric network

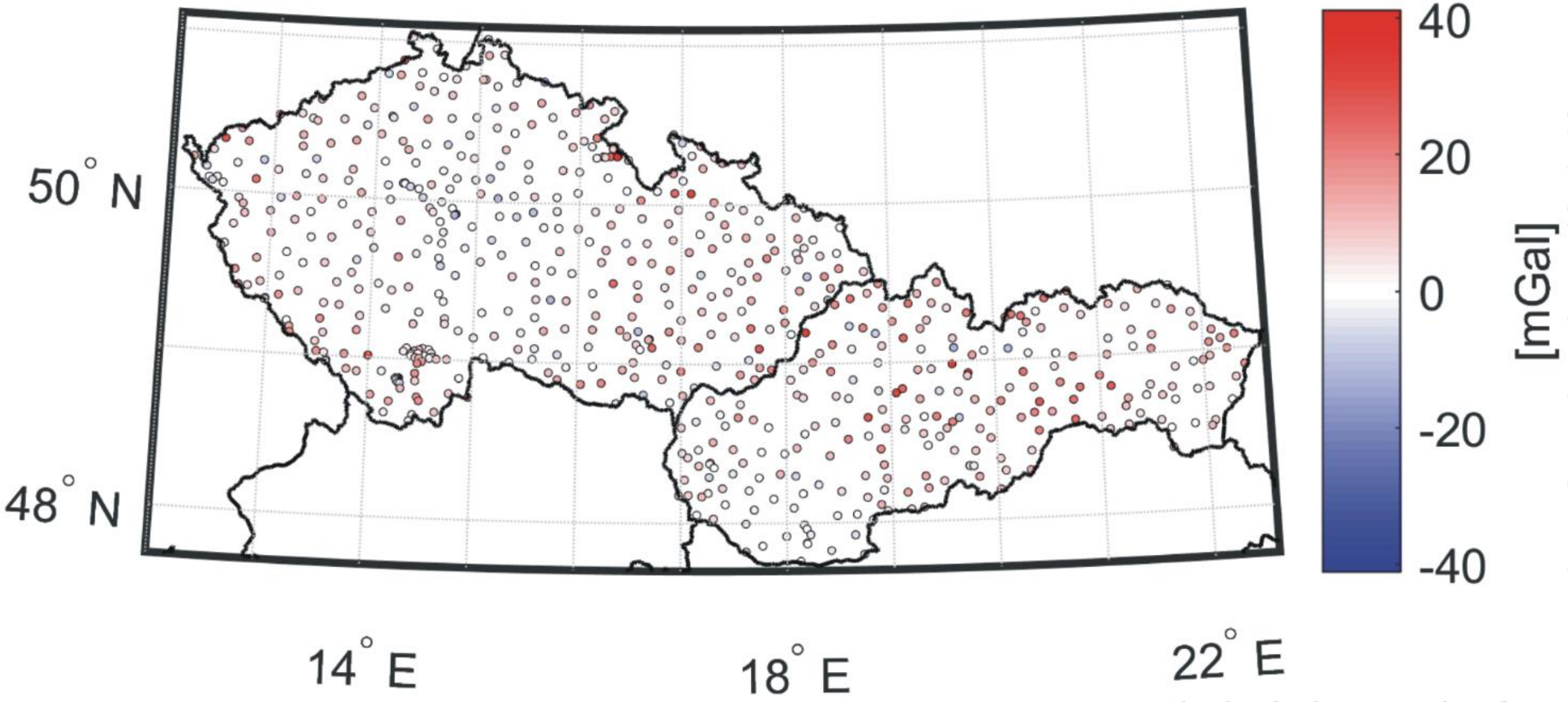


- The normal gravity was evaluated with respect to GRS80 reference ellipsoid

Results: $\Delta\delta g = \delta g^{ter} - \left(\delta g^{SDWC} + \delta g^{\text{EGM2008}}_{201-2190} + \delta g^{\text{dV\_ELL\_Earth2014\_5480}}_{2191-5480}\right)$

Differences (in mGal) between terrestrial gravity anomalies and gravity anomalies obtained from spectral combination method + EGM2008 + RTM (GOCE+ Geoexplore II grids).

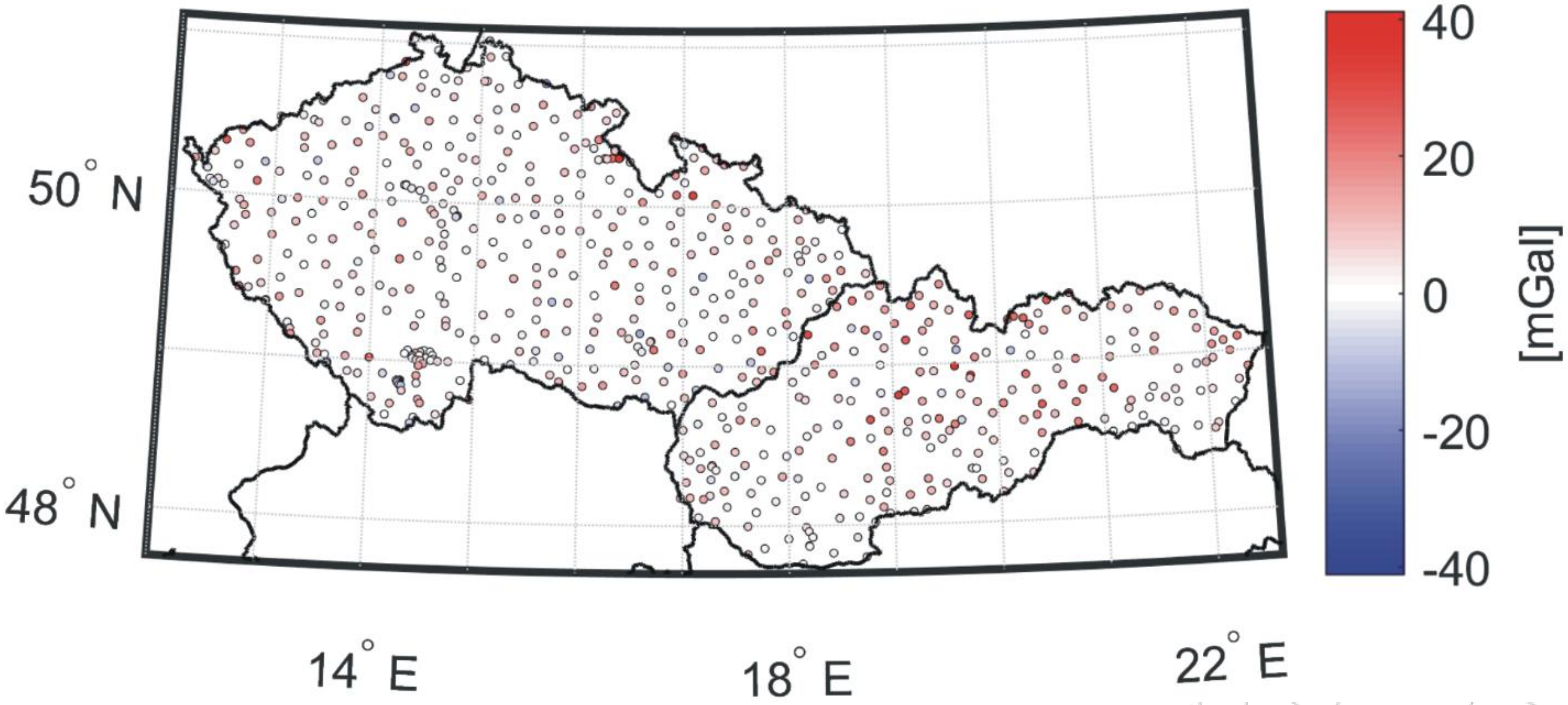
	VV	VH	HH	(VV,VH)	(VV,HH)	(VH,HH)	(VV,VH,HH)
std	8.284	8.108	8.114	8.108	8.114	8.114	8.114
max	39.841	38.907	39.033	38.904	39.050	39.041	39.042
min	-14.762	-14.495	-13.815	-14.498	-13.799	-13.808	-13.807
mean	6.252	6.332	6.446	6.329	6.462	6.453	6.454



Results: $\Delta\delta g = \delta g^{ter} - \left(\delta g^{SDWC} + \delta g^{\mathrm{EGM2008}}_{201-2190} + \delta g^{\mathrm{dV\_ELL\_Earth2014\_5480}}_{2191-5480}\right)$

Differences (in mGal) between terrestrial gravity anomalies and gravity anomalies obtained from spectral combination method + EGM2008 + RTM (Space-wise GOCE grids).

	VV	VH	HH	(VV,VH)	(VV,HH)	(VH,HH)	(VV,VH,HH)
std	7.873	7.885	7.873	7.885	7.873	7.873	7.873
max	40.929	41.142	41.048	41.142	41.048	41.049	41.049
min	-17.612	-17.624	-17.781	-17.624	-17.780	-17.779	-17.779
mean	6.294	6.312	6.295	6.312	6.295	6.296	6.296



Conclusion:

- we presented an alternative method for validation of GOCE-based gravitational gradients grids based on the spectral combination method,

- we validated two sets of grids, namely Space-wise GOCE grids and grids from the GOCE+Geoexplore II project,

- slightly better fit with respect to terrestrial data was achieved from Space-wise GOCE grids.

Thank you for your attention

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